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Dynamical Brane from Gravitational Dual of $\mathcal{N} = 2$ $Sp(N)$ Superconformal Field Theory

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ABSTRACT

The particular model of d5 higher derivative gravity which is dual to $\mathcal{N} = 2$ $Sp(N)$ SCFT is considered. (Perturbative) AdS Black Hole in such theory is constructed in the next-to-leading order of AdS/CFT correspondence. The surface counterterms are fixed by the conditions of well-defined variational procedure and of finiteness of AdS space (when brane goes to infinity). The dynamical brane is realized at the boundary of AdS Black Hole with the radius which is bigger than horizon radius. AdS/CFT correspondence dictates the parameters of gravitational dual in such a way that dynamical brane (observable Universe) always occurs outside the horizon.

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1 Introduction

In the modern studies of brane-worlds one assumes that observable Universe lies as a boundary in multidimensional bulk space where gravity on the brane is trapped [1] and bulk represents some AdS-like multidimensional background which follows from (IIB) string theory. The brane-world cosmology [2, 3] (and refs.therein) is usually constructed from (d5) AdS bulk space at the following conditions:
a. There are few free parameters of theory which are fine-tuned to get the desireable properties. In most cases these parameters are (negative) bulk cosmological constant and brane cosmological constant (brane tension). Playing with these two parameters one can construct various brane-worlds already in Einstein gravity (with surface terms). The coefficients of higher derivative terms in bulk (or brane) action may play the role of these parameters as well.
b. As bulk one chooses AdS space or its product with some other manifold.

Clearly, this is not dynamical mechanism for realization of observable brane Universe as parameters of theory are fine-tuned to get brane-world.

Presumably, the dynamically generated brane should be searched within AdS/CFT correspondence [4] where warped compactification introduces brane-world set-up. Indeed, one version of such scenario which is more suitable in frames of AdS/CFT correspondence has been presented in refs.[5] where brane tension is fixed from the beginning but quantum CFT living on the brane produces effective brane tension. This scenario is much more restrictive as brane-worlds are produced completely dynamically. Our approach lies in this line.

From another point, it is not clear what should be the bulk? It is expected that topology [6] should be very important in realization of warped compactification in string theory in frames of AdS/CFT correspondence. In recent letter [7] we considered the particular model of d5 higher derivative gravity which admits Schwarzschild-Anti de Sitter (S-AdS) black hole as exact bulk solution. It has been shown that four-dimensional brane (flat or de Sitter) could be realized dynamically. Hence, observable Universe may represent the boundary of higher dimensional AdS black hole.

In the present work we generalize the mechanism of ref.[7] and show that it may be realized completely within AdS/CFT correspondence. The starting point is the particular model of d5 HD gravity which appears as low-energy limit of compactificated IIB string theory. All parameters of the model are defined. The theory is expected to be dual to $\mathcal{N}=2$ SCFT with gauge group $Sp(N)$ [8] (see also [9]). This duality has been also checked by comparison of holographic and QFT conformal anomalies in refs.[10, 11] in the next-to-leading order of large-N expansion.

The surface counterterms are added to the starting action in frames of AdS/CFT correspondence. The parameters of these counterterms are not the arbitrary ones. They are fixed by two conditions:
a. The variational procedure should be well-defined.
b. The leading divergences of bulk AdS BH should be cancelled (finiteness). In particular, brane tension is fixed by these requirements.

Working with such gravitational dual of $\mathcal{N}=2$ SCFT we construct AdS BH perturbatively in the next-to-leading order of AdS/CFT correspondence. The (flat) brane is dynamically created as the boundary of such AdS BH and its radius which is bigger than horizon radius is found. The analogous AdS-like cosmological model is also briefly discussed.

2 AdS Black Hole with Dynamical Brane

Let us start from general action of higher derivative gravity. It is given by [4]:

$$S=\int d^{d+1}x\sqrt{-\hat{G}}\left\{a\hat{R}^2+b\hat{R}_{\mu\nu}\hat{R}^{\mu\nu}+c\hat{R}_{\mu\nu\xi\sigma}\hat{R}^{\mu\nu\xi\sigma}+\frac{1}{\kappa^2}\hat{R}-\Lambda\right\} . \tag{1}$$

The equations of motion derived from the above action (1) are:

$$\begin{aligned}0 \ = \ & -\frac{1}{2}\hat{G}_{\zeta\xi}\left\{a\hat{R}^2+b\hat{R}_{\mu\nu}\hat{R}^{(0)\mu\nu}+c\hat{R}_{\mu\nu\rho\sigma}\hat{R}^{\mu\nu\rho\sigma}+\frac{1}{\kappa^2}\hat{R}-\Lambda\right\} \\ & +2a\hat{R}\hat{R}_{\zeta\xi}+2b\hat{R}_{\mu\zeta}\hat{R}^{\mu}_{\xi}+2c\hat{R}_{\zeta\mu\nu\rho}\hat{R}_{\xi}^{\mu\nu\rho}+\frac{1}{\kappa^2}\hat{R}_{\zeta\xi} \\ & -a\left(D_\zeta D_\xi+D_\xi D_\zeta\right)\hat{R} \\ & +b\left(\hat{G}_{\zeta\xi}D_\rho D_\sigma\hat{R}^{\rho\sigma}-D_\xi D_\sigma\hat{R}^{\sigma}_{\zeta}-D_\zeta D_\sigma\hat{R}^{\sigma}_{\xi}+\Box\hat{R}_{\xi\zeta}\right)+4cD_\rho D_\kappa\hat{R}_{\zeta\ \xi}^{\ \rho\ \zeta} .\end{aligned} \tag{2}$$

[4]The conventions of curvatures are given by

$$R=g^{\mu\nu}R_{\mu\nu}, \quad R_{\mu\nu}=-\Gamma^{\lambda}_{\mu\lambda,\nu}+\Gamma^{\lambda}_{\mu\nu,\lambda}-\Gamma^{\eta}_{\mu\lambda}\Gamma^{\lambda}_{\nu\eta}+\Gamma^{\eta}_{\mu\nu}\Gamma^{\lambda}_{\lambda\eta}$$

$$R^{\lambda}_{\mu\nu\kappa}=\Gamma^{\lambda}_{\mu\kappa,\nu}-\Gamma^{\lambda}_{\mu\nu,\kappa}+\Gamma^{\eta}_{\mu\kappa}\Gamma^{\lambda}_{\nu\eta}-\Gamma^{\eta}_{\mu\nu}\Gamma^{\lambda}_{\kappa\eta}, \quad \Gamma^{\eta}_{\mu\lambda}=\frac{1}{2}g^{\eta\nu}\left(g_{\mu\nu,\lambda}+g_{\lambda\nu,\mu}-g_{\mu\lambda,\nu}\right) .$$

One assumes the following choice of metric

$$ds^2 = \hat{G_{\mu\nu}} dx^\mu dx^\nu = -\mathrm{e}^{2\rho} dt^2 + \mathrm{e}^{-2\rho} dr^2 + r^2 \sum_{i=1}^{d-1} \left(dx^i\right)^2 \ . \tag{3}$$

Here 4d part of metric is chosen to be flat only for simplicity. There is no problem to consider de Sitter or Anti-de Sitter 4d part of metric (only the corresponding calculations are technically a bit more involved).

If we further assume a, b, c are small compared with $\frac{1}{\kappa^2}$, we obtain the following AdS BH solution when $d+1=5$:

$$\mathrm{e}^{2\rho} = \frac{1}{r^2}\left\{-\mu + \left(1 + \frac{20a\kappa^2}{3} + \frac{4b\kappa^2}{3} + \frac{2c\kappa^2}{3}\right) r^4 + \frac{2c\kappa^2\mu^2}{r^4}\right\} \ . \tag{4}$$

As the leading order effect of a and b is simply to change the radius of AdS, we only consider the case $a=b=0$ in the following (Riemann curvature squared ones):

$$S = \int d^{d+1}x\sqrt{-\hat{G}}\left\{\frac{1}{\kappa^2}\left(\hat{R}+12\right) + c\hat{R}_{\mu\nu\xi\sigma}\hat{R}^{\mu\nu\xi\sigma}\right\} . \tag{5}$$

Especialy when $d+1=5$ and

$$\frac{1}{\kappa^2} = \frac{N^2}{4\pi^2} \ , \quad c = \frac{6N}{24\cdot 16\pi^2} \ , \tag{6}$$

the action (5) appears as the low-energy theory of type IIB string theory on $AdS_5 \times X^5$ where $X_5 = S^5/Z_2$ [8, 9]. This theory is expected to be dual to $\mathcal{N}=2$ theory with the gauge group $Sp(N)$.

When $a=b=0$, the perturbative solution in (4) looks like [12]:

$$\mathrm{e}^{2\rho} = \frac{1}{r^2}\left\{-\mu + \left(1+\frac{2}{3}\epsilon\right) r^4 + 2\epsilon\frac{\mu^2}{r^4}\right\}, \quad \epsilon \equiv c\kappa^2 \ . \tag{7}$$

In the following, we neglect the terms containing the higher powers of ϵ. If we assume the metric in the form of (3), the components of Ricci tensor and Riemann tensor are given by

$$\hat{R}_{tt} = \left(\rho'' + 2(\rho')^2 + \frac{(d-1)\rho'}{r}\right)\mathrm{e}^{4\rho}$$

$$
\begin{aligned}
\hat{R}_{rr} &= -\rho'' - 2(\rho')^2 - \frac{(d-1)\rho'}{r} \\
\hat{R}_{ij} &= (-2r\rho' - d + 2)\, e^{2\rho}\delta_{ij} \\
0 &= \text{other Ricci tensor components}
\end{aligned}
\tag{8}
$$

$$
\begin{aligned}
\hat{R}_{trtr} &= -\hat{R}_{trrt} = -\hat{R}_{rttr} = \hat{R}_{rtrt} \\
&= e^{2\rho}\left(\rho'' + 2\rho'^2\right) \\
\hat{R}_{titj} &= -\hat{R}_{ittj} = -\hat{R}_{tijt} = \hat{R}_{itjt} \\
&= r\rho'\delta_{ij} e^{4\rho} \\
\hat{R}_{rirj} &= -\hat{R}_{rijr} = -\hat{R}_{irrj} = \hat{R}_{irjr} \\
&= -r\rho'\delta_{ij} \\
\hat{R}_{ijkl} &= -r^2 e^{2\rho}\left(\delta_{ik}\delta_{jl} - \delta_{il}\delta_{jk}\right) \\
0 &= \text{other Riemann tensor components} \quad .
\end{aligned}
\tag{9}
$$

The scalar curvature and the square of Riemann tensor are

$$
\begin{aligned}
\hat{R} &= \left(-2\rho'' - 4(\rho')^2 - \frac{4(d-1)\rho'}{r} - \frac{(d-2)(d-1)}{r^2}\right) e^{2\rho} \\
\hat{R}_{\mu\nu\xi\sigma}\hat{R}^{\mu\nu\xi\sigma} &= 4\hat{R}_{trtr}\hat{R}^{trtr} + 4\hat{R}_{titj}\hat{R}^{titj} + 4\hat{R}_{rirj}\hat{R}^{rirj} + \hat{R}_{ijkl}\hat{R}^{ijkl} \\
&= 4e^{4\rho}\left(\rho'' + 2\rho'^2\right)^2 + 4(d-1)r^{-2}\rho'^2 e^{4\rho} + 4(d-1)r^{-2}\rho'^2 e^{4\rho} \\
&\quad +2(d-1)(d-2)r^{-4}e^{4\rho} \\
&= 4e^{4\rho}\left(\rho'' + 2\rho'^2\right)^2 + 8(d-1)r^{-2}\rho'^2 e^{4\rho} + 2(d-1)(d-2)r^{-4}e^{4\rho}.
\end{aligned}
\tag{10}
$$

One supposes that there is a boundary at $r = r_0$, where brane lies. Then we need to add boundary term, especially 4 dimensional cosmological term in order to get brane-world Universe, i.e. RS scenario for 4d gravity [1]. Usually such surface term is chosen to be arbitrary. Its fine-tuning is responsible for creation of brane-world.

In our scenario (within AdS/CFT correspondence) the surface counterterms are not arbitrary. Such surface term makes the variational principle to be well-defined and complete AdS space to be finite when brane goes to infinity. One takes the surface terms in the following form [13]:

$$
S_b = S_b^{(1)} + S_b^{(2)} \tag{11}
$$

$$
S_b^{(1)} = \int d^4x\sqrt{\hat{g}}\left[4\tilde{a}\hat{R}D_\mu n^\mu + 2\tilde{b_1} n_\mu n_\nu \hat{R}^{\mu\nu} D_\sigma n^\sigma + 2\tilde{b_2}\hat{R}_{\mu\nu}D^\mu n^\nu\right.
$$

$$
\begin{aligned}
&\quad +8\tilde{c}n_\mu n_\nu \hat{R}^{\mu\tau\nu\sigma}D_\tau n_\sigma + \frac{2}{\tilde{\kappa}^2}D_\mu n^\mu\Big] \\
S_b^{(2)} &= -\eta\int d^4x\sqrt{\hat{g}}
\end{aligned}
$$

where the normal vector n^μ and its covariant derivatives are given by

$$
\begin{aligned}
n^r &= \mathrm{e}^\rho, \text{other components} = 0 \\
D_r n^r &= 0,\ D_t n^t = \mathrm{e}^\rho \rho',\ D_i n^j = \frac{\mathrm{e}^\rho}{r}\delta_i^j \\
D_\mu n^\mu &= \mathrm{e}^\rho\rho' + \frac{(d-1)\mathrm{e}^\rho}{r}\ .
\end{aligned}
\tag{12}
$$

In (11), we can choose $\tilde{b}_1 = \tilde{b}_2$ but as we will see later, it is convienient to treat them as independent parameters when one considers the black hole background as in [7].

When we substitute the solution (7) into the bulk action (5) with $d = 4$, there appears the divergence if there is no brane, which can be a boundary of the spacetime. In order to regularize the divergence, we restrict the integration region of r to be finite $\int d^5x \to \int d^4x \int_0^r dr$ and we assume the surface terms in (11) appear on the boundary. The parameter η in (11) is determined by the condition that the corresponding term should cancell the leading divergence. As one finds after the integration on r,

$$
S \sim \frac{r^4}{\kappa^2}\left(-2 + \frac{20}{3}\epsilon\right)\int d^4x + o\left(r^4\right) \tag{13}
$$

and the surface terms in (11) behave, when r is large, as follows:

$$
\begin{aligned}
S_b &\sim r^4\left\{-320\tilde{a} - 32\tilde{b}_1 - 32\tilde{b}_2 - 32\tilde{c} + \frac{8}{\tilde{\kappa}^2}\left(1+\frac{2}{3}\epsilon\right)\right. \\
&\quad \left. -\eta\left(1+\frac{1}{3}\epsilon\right)\right\}\int d^4x + o\left(r^4\right)\ .
\end{aligned}
\tag{14}
$$

Then one gets

$$
\eta = \frac{1}{\kappa^2}\left(-2+6\epsilon\right) - 320\tilde{a} - 32\tilde{b}_1 - 32\tilde{b}_2 - 32\tilde{c} + \frac{8}{\tilde{\kappa}^2}\left(1+\frac{1}{3}\epsilon\right)\ . \tag{15}
$$

The variation of the action (5) and (11) on the boundary which lies at $r = r_0$ gives

$$
\delta S|_{r=r_0} = \int d^4x r_0^{d-1}\mathrm{e}^{2\rho}
$$

$$
\begin{aligned}
&\times \left[\frac{1}{\kappa^2}\left\{-2\delta\rho' + \delta\rho\left(-8\rho' - \frac{4(d-1)}{r_0}\right)\right\}\right.\\
&\left. + c\mathrm{e}^{2\rho}\left\{8(\rho'' + 2(\rho')^2)(\delta\rho' + 4\rho'\delta\rho) + \frac{16(d-1)}{r_0^2}\rho'\delta\rho\right\}\right] \qquad (16)
\end{aligned}
$$

$$
\begin{aligned}
\delta S_b \;=\; & \int d^4x r_0^{d-1}\mathrm{e}^{\rho}\\
\times\; & \left[\mathrm{e}^{3\rho}\delta\rho''\left\{\left(-8\tilde{a} - 2\tilde{b}_1\right)\left(\rho' + \frac{d-1}{r_0}\right) - 2\tilde{b}_2\rho' - 8\tilde{c}\rho'\right\}\right.\\
& +\delta\rho'\left\{\tilde{a}\left\{\left(-32\rho' - \frac{16(d-1)}{r_0}\right)\left(\rho' + \frac{d-1}{r_0}\right)\mathrm{e}^{3\rho}\right.\right.\\
& +4\left(\left(-2\rho'' - 4\left(\rho'\right)^2 - \frac{4(d-1)\rho'}{r_0} - \frac{(d-2)(d-1)}{r_0^2}\right)\mathrm{e}^{2\rho}\right.\\
& \left.\left. +\frac{(d-1)k}{r_0^2}\right)\mathrm{e}^{\rho}\right\}\\
& +\tilde{b}_1\left\{\left(-8\rho' - \frac{2(d-1)}{r_0}\right)\left(\rho' + \frac{d-1}{r_0}\right)\mathrm{e}^{3\rho}\right.\\
& \left. +2\left(-\rho'' - 2\left(\rho'\right)^2 - \frac{(d-1)\rho'}{r_0}\right)\mathrm{e}^{3\rho}\right\}\\
& +\tilde{b}_2\left\{2\left(-\rho'' - 2\left(\rho'\right)^2 - \frac{(d-1)\rho'}{r_0}\right)\mathrm{e}^{3\rho}\right.\\
& \left. -8\left(\rho'\right)^2\mathrm{e}^{3\rho} - \mathrm{e}^{3\rho}\rho'\frac{2(d-1)}{r_0} - \mathrm{e}^{3\rho}\frac{4(d-1)}{r_0^2}\right\} + \frac{2\mathrm{e}^{\rho}}{\tilde{\kappa}^2}\\
& \left. +8\tilde{c}\mathrm{e}^{3\rho}\left(-6\left(\rho'\right)^2 - \rho'' - \frac{(d-1)}{r_0^2}\right)\right\}\\
& +\delta\rho\left\{4\tilde{a}\left\{4\mathrm{e}^{3\rho}\left(-2\rho'' - 4\left(\rho'\right)^2 - \frac{4(d-1)\rho'}{r_0}\right.\right.\right.\\
& \left.\left. -\frac{(d-2)(d-1)}{r_0^2}\right)\left(\rho' + \frac{d-1}{r_0}\right) + \frac{2\mathrm{e}^{\rho}(d-1)k}{r_0^2}\left(\rho' + \frac{d-1}{r_0}\right)\right\}\\
& +8\tilde{b}_1\left(-\rho'' - 2\left(\rho'\right)^2 - \frac{(d-1)\rho'}{r_0}\right)\left(\rho' + \frac{d-1}{r_0}\right)\mathrm{e}^{3\rho}\\
& +2\tilde{b}_2\left\{4\rho'\left(-\rho'' - 2\left(\rho'\right)^2 - \frac{(d-1)\rho'}{r_0}\right)\mathrm{e}^{3\rho}\right.
\end{aligned}
$$

$$
\begin{aligned}
&+\frac{4(d-1)}{r_0^3}\left(-2r\rho' - d + 2\right)\mathrm{e}^{3\rho} \\
&\left.+\frac{2k(d-1)}{r^3}\right\} + \frac{4\mathrm{e}^{\rho}}{\tilde{\kappa}^2}\left(\rho' + \frac{d-1}{r_0}\right) \\
&\left.\left.-32\tilde{c}\mathrm{e}^{3\rho}\left((\rho'' + 2\rho'^2)\rho' + \frac{\rho'(d-1)}{r_0^2}\right) - \eta\right\}\right] \ .
\end{aligned} \tag{17}
$$

To satisfy the condition that variational principle in the theory under discussion is well-defined, the coefficients of $\delta\rho''$ and $\delta\rho'$ must vanish. From the condition that the coefficient of $\delta\rho''$ must vanish, one gets

$$
\tilde{b}_1 = -4\tilde{a}, \quad \tilde{b}_2 = -4\tilde{c} \ . \tag{18}
$$

Then substituting the solution (7) and the condition (18), we find (putting $d = 4$)

$$
\begin{aligned}
&\delta S + \delta S_b \\
&= \int d^4x r_0^3 \left[\delta\rho'\left(2\left(\frac{1}{\tilde{\kappa}^2} - \frac{1}{\kappa^2}\right) + 8c\left(-3r_0^{-4}\mu + 1\right)\right.\right. \\
&\quad -12\tilde{a}(8r_0^{-4}\mu + 12(1 - \mu r_0^{-4})) \\
&\quad \left. +24\tilde{c}(3 + r_0^{-4}\mu)\right)\mathrm{e}^{2\rho} \\
&\quad +\delta\rho\left(-\frac{4}{\kappa^2}\left\{-\mu r_0^{-3} + 5\left(1 + \frac{2}{3}\epsilon\right)r_0 - 6\epsilon\mu^2 r_0^{-7}\right\}\right. \\
&\quad +\frac{4}{\tilde{\kappa}^2}\left\{-2r_0^{-3}\mu + 4\left(1 + \frac{2}{3}\epsilon\right)r_0\right\} \\
&\quad +16c\left\{-9\mu^2 r_0^{-7} - 4r_0^{-3}\mu + 5r_0\right\} \\
&\quad -48\tilde{a}\left\{-8r_0^{-3}\mu + 16r_0\right\} \\
&\quad +96\tilde{c}\left\{2\mu^2 r_0^{-7} - 2\mu r_0^{-3} + 4r_0\right\} \\
&\quad \left.\left.-\eta\frac{\sqrt{-\mu + r_0^4}}{r_0}\left\{1 + \frac{\epsilon r_0^4}{3\left(-\mu + r_0^4\right)} - \frac{\epsilon\mu^2}{r_0^4\left(-\mu + r_0^4\right)}\right\}\right)\right]
\end{aligned} \tag{19}
$$

The condition that the coefficient of $\delta\rho'$ vanishes gives $\tilde{a}, \tilde{c}$ as

$$
\begin{aligned}
\tilde{a} &= \frac{1}{144}\left(40c + \left(\frac{1}{\tilde{\kappa}^2} - \frac{1}{\kappa^2}\right)\right) \\
\tilde{c} &= \frac{4}{9}c - \frac{1}{72}\left(\frac{1}{\tilde{\kappa}^2} - \frac{1}{\kappa^2}\right) \ .
\end{aligned} \tag{20}
$$

Substituting above $\tilde{a}, \tilde{c}$ and $c = \frac{\epsilon}{\kappa^2}$, the condition that the coefficient of $\delta\rho$ vanishes leads to:

$$
\begin{aligned}
0 &= F(r_0) \\
&\equiv \frac{1}{\tilde{\kappa}^2}\left\{-\frac{8}{3}\mu^2 r_0^{-7} - \frac{8}{3}\mu r_0^{-3} + \frac{16}{3}(1+2\epsilon)r_0\right\} \\
&\quad + \frac{1}{\kappa^2}\left\{-\frac{8}{3}(13\epsilon - 1)\mu^2 r_0^{-7} - \frac{4}{3}(32\epsilon+1)\mu r_0^{-3} + \left(24\epsilon - \frac{28}{3}\right) r_0\right\} \\
&\quad - \eta\frac{\sqrt{-\mu + r_0^4}}{r_0}\left\{1 + \frac{\epsilon r_0^4}{3\left(-\mu + r_0^4\right)} - \frac{\epsilon\mu^2}{r_0^4\left(-\mu + r_0^4\right)}\right\}
\end{aligned} \tag{21}
$$

Using (15), (18) and (20), we find η in (21) has the following form:

$$
\eta = \frac{2}{3\kappa^2}(1 - 7\epsilon) + \frac{16}{3\tilde{\kappa}^2}\left(1 + \frac{1}{2}\epsilon\right) . \tag{22}
$$

Hence, the coefficients of surface counterterms in AdS/CFT correspondence are fixed now.

Eq.(21) can be regarded as the equation to determine r_0, i.e., where brane lies. When $r_0 \to \infty$, $F(r_0)$ behaves as

$$
\begin{aligned}
F(r_0) &\sim \left\{\frac{1}{\tilde{\kappa}^2}\left(\frac{16}{3} + \frac{32}{3}\epsilon\right) + \frac{1}{\kappa^2}\left(-\frac{28}{3} + 24\epsilon\right) - \eta\left(1 + \frac{\epsilon}{3}\right)\right\} r_0 \\
&\sim -\frac{10}{\kappa^2} + \mathcal{O}(\epsilon) < 0 .
\end{aligned} \tag{23}
$$

On the other hand, when $r_0 \to \mu^{\frac{1}{4}}$, $F(r_0)$ behaves as

$$
F(r_0) \sim \frac{2}{3}\frac{\eta\epsilon\mu}{\sqrt{-\mu + r_0^4}} . \tag{24}
$$

In case of the string theory dual to $\mathcal{N} = 2$ theory with the gauge group $Sp(N)$ in (6), c and therefore ϵ are positive. Combining (23) and (24), we find that there is a solution r_0 satisfying the brane equation (21) in the $\mathcal{N} = 2$ SCFT case. Equations (23) and (24) tell that $r_0^4 - \mu = \mathcal{O}\left(\epsilon^2\right)$. Then assuming

$$
r_0^4 = \mu + \alpha^2\epsilon^2 + \mathcal{O}\left(\epsilon^3\right) \quad (\alpha > 0) \tag{25}
$$

and substituting (22) and (25) into (21), one finds

$$\alpha = \frac{2}{3} \cdot \frac{1 + \frac{\tilde{\kappa}^2}{8\kappa^2}}{1 + \frac{13\tilde{\kappa}^2}{8\kappa^2}} + \mathcal{O}(\epsilon) \ . \tag{26}$$

Especially if we choose $\tilde{\kappa}^2 = \kappa^2$ as in the original Gibbons-Hawking term[14] , we get

$$\alpha = \frac{2}{7} \tag{27}$$

that is

$$r_0^4 = \mu + \left(\frac{2}{7}\epsilon\right)^2 + \mathcal{O}\left(\epsilon^3\right) \ . \tag{28}$$

One should note that solution r_0 in (26) (with (25)) or (28) is larger than the unperturbative horizon which lies at $r = \mu^{\frac{1}{4}}$ (in terms of mass of AdS BH under consideration). As one sees, c-correction makes the radius of the horizon smaller if c or ϵ is positive. Then the brane always exists outside the horizon. In other words, AdS/CFT duality predicts the correct signs of gravitational action in such a way that observable Universe is realized as the brane outside of multi-dimensional BH horizon! The whole Universe evolution could occur within less than one period of BH time.

Let us consider the thermodynamical quantities. In the solution in (7), the radius r_h of the horizon and the temperature T are given by

$$r_h \equiv \mu^{\frac{1}{4}}\left(1 - \frac{2}{3}\epsilon\right) \ , \quad T = \frac{\mu^{\frac{1}{4}}}{\pi}(1 - 2\epsilon) = \frac{\mu^{\frac{1}{4}}}{\pi}\left(1 - \frac{1}{8N}\right) \ . \tag{29}$$

After Wick-rotating the time variables by $t \rightarrow i\tau$, the free energy $\mathcal{F}$ can be obtained from the action S in (1) with $a = b = 0$ where the classical solution is substituted:

$$\mathcal{F} = \frac{1}{T} S \ . \tag{30}$$

Using (2) with $a = b = 0$, (6) and (7), one finds

$$\begin{aligned} S &= \frac{N^2}{4\pi^2}\int d^5x\sqrt{g}\left\{8 - \frac{2\epsilon}{3}\left(40 + \frac{72\mu^2}{r^8}\right)\right\} \\ &= \frac{N^2 V_3}{4\pi^2 T}\int_{r_h}^{\infty} dr r^3 \left\{8 - \frac{2\epsilon}{3}\left(40 + \frac{72\mu^2}{r^8}\right)\right\} \ . \end{aligned} \tag{31}$$

Here V_3 is the volume of 3d flat space and we assume τ has a period of $\frac{1}{T}$. The expression of S contains the divergence coming from large r. In order to subtract the divergence, we regularize S in (31) by cutting off the integral at a large radius $r_{\rm max}$ and subtracting the solution with $\mu = 0$:

$$\begin{aligned} S_{\rm reg} &= \frac{N^2 V_3}{4\pi^2 T}\left(\int_{r_h}^{\infty} dr r^3 \left\{8 - \frac{2\epsilon}{3}\left(40 + \frac{72\mu^2}{r^8}\right)\right\}\right. \\ &\quad \left. - \mathrm{e}^{\rho(r=r_{\rm max}) - \rho(r=r_{\rm max};\mu=0)} \int_0^{r_{\rm max}} dr r^3\right)\left\{8 - \frac{80\epsilon}{3}\right\} . \end{aligned} \tag{32}$$

The factor $\mathrm{e}^{\rho(r=r_{\rm max}) - \rho(r=r_{\rm max};\mu=0)}$ is chosen so that the proper length of the circle which corresponds to the period $\frac{1}{T}$ in the Euclidean time at $r = r_{max}$ coincides with each other in the two solutions. Then we find

$$\mathcal{F} = -\frac{N^2 V_3 (\pi T)^4}{4\pi^2}\left(1 + \frac{3}{4N}\right) . \tag{33}$$

The entropy $\mathcal{S}$ and the mass (energy) E are given by

$$\begin{aligned} \mathcal{S} &= -\frac{d\mathcal{F}}{dT} = \frac{N^2 V_3 (\pi T)^4}{\pi^2 T}\left(1 + \frac{3}{4N}\right) \\ E &= \mathcal{F} + T\mathcal{S} = \frac{3N^2 V_3 (\pi T)^4}{4\pi^2}\left(1 + \frac{3}{4N}\right) . \end{aligned} \tag{34}$$

Hence, as is expected the presence of non-trivial boundary does not influence BH thermodynamics [12] which gives the corresponding description for dual SCFT at finite temperature in the next-to-leading order of large-N expansion.

3 Discussion

As we demonstrated on the example of gravitational dual (HD gravity) of SCFT with two supersymmetries the dynamical brane (observable Universe) may occur as the boundary of d5 AdS BH in the next-to-leading order of AdS/CFT correspondence. The coefficients of surface counterterms are consistently fixed within AdS/CFT correspondence, they are not fine-tuned by condition of existence of brane (which is the usual case in brane-world scenarios). Moreover, the signs of coefficients (predicted by AdS/CFT) of HD gravity are such that brane radius is bigger than horizon radius. In other words, observable Universe may be realized as the boundary of multi-dimensional AdS BH but outside of horison. It could be interesting to develop further the details of such scenario.

Few remarks are in order. Inside the horizon $r < r_h$, if one renames r as t and t as r we obtain a metric corresponding to AdS-like cosmological model:

$$\begin{aligned} ds^2 &= -\mathrm{e}^{2\rho}dt^2 + \mathrm{e}^{-2\rho}dr^2 + r^2\sum_{i=1}^{d-1}\left(dx^i\right)^2 \ , \\ \mathrm{e}^{-2\rho} &= -\frac{1}{t^2}\left\{-\mu + \left(1+\frac{2}{3}\epsilon\right)t^4 + 2\epsilon\frac{\mu^2}{t^4}\right\} \ . \end{aligned} \tag{35}$$

The leading behavior of the curvature is the same as in the black hole case with $c = 0$ and we find that there is a curvature singularity at $t = 0$:

$$\hat{R}_{\mu\nu\xi\sigma}\hat{R}^{\mu\nu\xi\sigma} = 40 + \frac{72\mu^2}{t^8} \ . \tag{36}$$

and there is a horizon at $t = r_h$. The singularity might be regarded as a kind of big bang. The topology of the spatial part is $\mathrm{S}_1\times\mathrm{R}_3$ if we impose a periodic boundary condition on r or $\mathrm{R}\times\mathrm{R}_3$ if not. Here R_3 corresponds to the coordinates x^i.

We can consider the free energy analogue of $\mathcal{F}^{\cos}$ when $c = 0$ as follows:

$$T\mathcal{F}^{\cos} = \frac{8N^2V_3}{4\pi^2 T}\int_0^{r_h} dt\, t^3 = \frac{2N^2V_3\left(\pi T\right)^4}{4\pi^2 T} \ . \tag{37}$$

If $c \neq 0$, however, the free energy diverges due to the singularity of $\hat{R}_{\mu\nu\xi\sigma}\hat{R}^{\mu\nu\xi\sigma}$ in (36) when $t \to 0$. This simple consideration indicates on some duality between BH solutions and cosmological solutions in multidimensional HD gravity. As there exists well-developed technique to study the cosmological models, the above trick may be useful in the investigation of BH interior.

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